Formal Logic

Lecture 1: The Syntax of Propositional Logic

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Introduction

Polling app

- To participate in this interactive lecture please download:

  the **Poll Everywhere** app

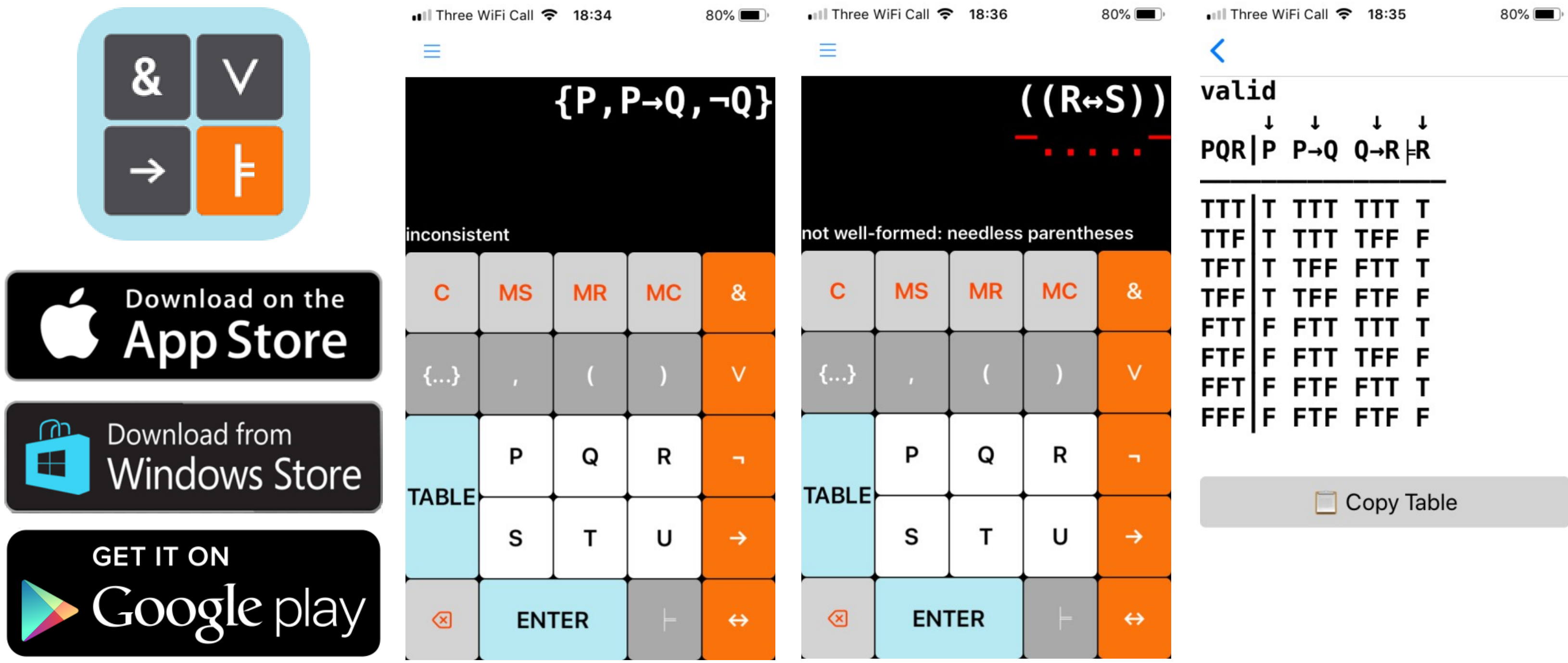
  available for iOS and Android

  Once downloaded join our poll using the name:

  PollEv.com/ioannisvotsi184

Logic app

- The Logic Calculator is a free app that I created to perform propositional logic operations.



- I plan to release more advanced versions that allow for derivations, predicate logic, etc.

Semester plan

WEEK 1: The Syntax of Propositional Logic
WEEK 2: The Semantics of Propositional Logic
WEEK 3: Solving Exercises in Class: Set 1
WEEK 4: Derivations in Propositional Logic
WEEK 5: Solving Exercises in Class: Set 2
WEEK 6: The Syntax of Predicate Logic with Identity
WEEK 7: READING WEEK
WEEK 8: Sets and Relations
WEEK 9: The Semantics of Predicate Logic with Identity
WEEK 10: Solving Exercises in Class: Set 5
WEEK 11: Mock Exam II

NB: Exercise sets 3 and 4 make up your formative assignments!

Deductive Logic: The Basic Ideas

Arguments

- Logic is the formal study of arguments. An *argument* consists of *one or more premises** and a *conclusion*.

  NB: When we speak about logic, what is typically meant is *deductive* logic, a.k.a. ‘classical logic’.

- Premises and conclusions are declarative sentences. We want to find out whether arguments are **valid**.

- In short, whether the conclusion **follows** from the premises.

- That is, whether the conclusion is a **logical consequence** of the premises.

1. Premise 1
2. Premise 2
…
n. Premise n

∴ Conclusion

Sentences

- The sentences or propositions (unless otherwise noted I will use these interchangeably) in question are declarative.

- Compare:

The mat is on the floor.	[declarative]
Put the mat on the floor.	[imperative]
Is the mat on the floor?	[interrogative]

NB: Hereafter, and unless otherwise noted, whenever we talk about sentences, we mean declarative sentences.

Deduction: Validity and soundness

- An argument is ***valid*** if and only if the truth of the premises guarantees the truth of the conclusion.

  NB: The definition should be read 'If the premises *were* true, the conclusion *would also* be true'.

- Here's another formulation:

  An argument is ***valid*** iff and only if under no interpretation the premises are true and the conclusion false.

- An argument is ***sound*** if and only if it is valid and its premises are true.

The formal in logic

- I said earlier that logic is the formal study of arguments. What do we mean by formal?

- It focuses on the abstract form of arguments and, as such, doesn't require subject-specific knowledge.

Example:

1. All Schlorgs are bfine.
2. Muement is a Schlorg.

∴ Muement is bfine.

Risk and novelty

- If the truth of the premises *guarantees* the truth of the conclusion, what does this tell us about deduction?

  Advantage: It is the safest form of inference as it eliminates all risk. It is thus known as 'truth-/content-preserving'.

  Disadvantage: It produces no *new* content.

- After all, the content of the conclusion is already included in the content of the premises.

- Lacking logical omniscience, one can of course learn (or better yet realise) that some content is part of the premises.

Truth/content preservation

Premises Conclusion

Examples

1. Everest is the highest mountain on Earth.
2. Mariana is the deepest trench on Earth.

∴ Everest is the highest mountain on Earth and Mariana is the deepest trench on Earth.

1. If it rains, the dam will burst.
2. It rained.

∴ The dam burst.

Validity ≠ truth

- All sorts of arguments with one or more false premises and even a false conclusion are valid.

  1. All human beings are benevolent.
  2. All benevolent beings are idiots.

  ∴ All human beings are idiots.

- Also, arguments with true premises and a true conclusion may be invalid.

  Queen Elizabeth II was born in 1926.

  ∴ Queen Elizabeth II is 1.63 meters tall.

Validity and logical truth

- Even arguments with *no* premises may in fact be valid.

$\therefore$ All humans are humans

- This is the case when the conclusion is a logical truth (more commonly known as a tautology).

  A sentence is **logically true** *if and only if* it is true under any interpretation.

- Such arguments are valid because no circumstances exist where the premises are true and the conclusion is false.

Monotonicity

- Deductive logic is monotonic.

  Monotonicity: If an argument is valid, it remains valid no matter how many or what premises we add.

- What if we add a false statement?

  Reply: That doesn't matter as validity does not require that the premises are true!

- What if we add a contradiction?

  Reply: Again, it doesn't matter because in classical deductive logic, anything follows from a contradiction!

Validity and invalidity is applied to

propositions

sets of propositions

arguments

systems of logic

Deductive logic is ______ and ______.

truth-increasing, risk-free

truth-preserving, risk-free

truth-increasing, risky

truth-preserving, risky

An argument is valid if and only if...

The Propositional Calculus

Propositional vs. predicate logic

DEDUCTIVE LOGIC

PROPOSITIONAL LOGIC

Formal study of arguments at the coarse-grained level of whole sentences.

NB: A.k.a. ‘propositional calculus’, ‘sentential logic’.

PREDICATE LOGIC

Formal study of arguments at the sub-sentential (and thus more fine-grained) level of sentences.

NB: A.k.a. ‘first-order logic’, ‘predicate calculus’.

The syntax of L_1

- Propositional logic employs an artificial language which we can call ‘L_1’.

- All languages have a syntax. Their syntax uses rules to specify how to form expressions in that language.

  Example: In English, one such rule is: Subject-Verb-Object. For instance, ‘John hates cricket’.

- Note that this order is independent of the specific meanings (a.k.a. the semantics) of ‘John’, ‘hates’ and ‘cricket’.

- Similarly, the expressions of the formal language of propositional logic, call it ‘L_1’, are also specified by rules.

The language(s)

- Strictly speaking, we are employing two languages here:

  1) The (object) language L_1 that we use to actually do logic.

  2) The (meta) language that we use to describe L_1.

  NB: Hereafter, we ignore the meta-language for simplicity.

- The language of propositional logic consists of

  non-logical terms: **letters** denoting **sentences**, e.g. P, Q and R.

  logical terms: **symbols** denoting **logical connectives**, e.g. ¬, ∧, ∨, →, ↔.

Logical connectives

- The meaning of these connectives corresponds *roughly* to the following natural language expressions:

	SYMBOL	NAME	ENGLISH 'EQUIVALENT'
unary	$\neg$ (also $\sim$)	negation	not
binary	& (also $\wedge$)	conjunction	and
	$\vee$ (also +)	disjunction	or
	$\rightarrow$ (also $\supset$)	conditional	if... then...
	$\leftrightarrow$ (also $\equiv$)	bi-conditional	...if and only if...

Logical connectives

- The meaning of these connectives corresponds *roughly* to the following natural language expressions:

	SYMBOL	NAME	EXAMPLES
unary	$\neg$ (also $\sim$)	negation	$\neg P$, $\neg Q$, $\neg(P \,\&\, Q)$
binary	& (also $\wedge$)	conjunction	$P \,\&\, Q$, $(Q \vee R) \,\&\, P$
	$\vee$ (also +)	disjunction	$P \vee Q$, $\neg(P \vee Q)$
	$\rightarrow$ (also $\supset$)	conditional	$P \rightarrow Q$, $P \,\&\, (Q \rightarrow R)$
	$\leftrightarrow$ (also $\equiv$)	bi-conditional	$P \leftrightarrow Q$, $P \leftrightarrow (R \vee Q)$

Atomic vs. compound sentences

- Atomic sentences *cannot* be decomposed into other sentences and are denoted by single letters, e.g. P, Q and R.

- Complex or compound sentences are decomposable. They consist of sentences joined by connectives.

  Examples:

  $\neg R$
  $P \& Q$
  $P \rightarrow P$
  $(\neg Q \vee P) \& R$

Well-formed formulae

- Definition of a **sentence (a.k.a. well-formed formula)** in L_1:

  1. All sentence letters A, B, C… are sentences of L_1.
  2. If A, B are sentences of L_1, then $\neg A$, $(A \& B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \leftrightarrow B)$ are sentences of L_1.
  3. Nothing else is a sentence of L_1.

  Examples of well-formed formulae (a.k.a. sentences):

  $\neg A$ $(A \& A)$ $(A \& (B \vee C))$ $((D \rightarrow B) \leftrightarrow (\neg\neg C \rightarrow G))$

  Examples of formulae that are *not* well-formed:

  $\neg \& A$ $((A \& B))$ $(A \& \& B)$ $(A \rightarrow B)C$

  NB1: An infinite number of sentences can be constructed.

Well-formed formulae

- Definition of a **sentence (a.k.a. well-formed formula)** in L_1:

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  2. If A, B are sentences of L_1, then ¬A, (A&B), (A∨B), (A→B) and (A↔B) are sentences of L_1.
  3. Nothing else is a sentence of L_1.

  Examples of well-formed formulae (a.k.a. sentences):

¬A	(A&A)	(A&(B∨C))	((D→B)↔(¬¬C→G))
	A&A	A&(B∨C)	(D→B)↔(¬¬C→G)

  Examples of formulae that are *not* well-formed:

¬&A	((A&B))	(A&&B)	(A →B)C

  NB1: An infinite number of sentences can be constructed.
  NB2: By convention, we can drop the outermost brackets.

Commutativity

- **Binary connective:** / **Name of components related**:

Binary connective	Name of components related
conjunction	conjuncts
disjunction	disjuncts
conditional	antecedent, consequent
bi-conditional	---------

- Reordering the propositions related changes the meaning of the whole proposition only in the case of conditionals.

(A&B)	is the same as	(B&A)
(A∨B)	is the same as	(B∨A)
($A \leftrightarrow B$)	is the same as	($B \leftrightarrow A$)
($A \rightarrow B$)	is not the same as	($B \rightarrow A$)

NB: Akin to math (+, × are commutative but –, ÷ are not).

Consistency

- A set of sentences is denoted by braces/curly brackets, e.g. $\{P, P \rightarrow Q, \neg R\}$.

- Consistency is a property of sets of sentences.

  A set of sentences is **consistent** if and only if they *can* all be true at the same time.

- Examples:
  {Donald has children, Donald does not have children}
  {Jeremy's car is black, Boris' cat is 10 years old}
  {Either Tim or Liz will go to the party, Neither Tim nor Liz will go to the party}
  {If I am in Japan then I am not in England, I am in England, I am not in Japan}

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Which of the following is a unary connective?

disjunction

conditional

conjunction

negation

Which of the following is well-formed?

$\neg P(\&Q)$

$\neg\neg\neg P\&P$

$Q\neg \vee P$

$(A\&(B))$

Devise three well-formed formulae and share them with the class.

The End